## Appendix A

This appendix includes the solutions and answers to end of segment selfassessment problems and questions.

## Segment 1 - Solutions

1. In an AC system, a voltage source $V(t)=120 \operatorname{Sin}\left(377 t+0^{\circ}\right)$ volts, rms, sets up a current of $\mathbf{I}(\mathbf{t})=\mathbf{5} \operatorname{Sin}\left(\mathbf{3 7 7 t}+\mathbf{4 5}^{\circ}\right)$ amps, rms. Calculate the maximum values of voltage and current in this case.

## Solution:

According the Eq. 1.2:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{p}}=\mathrm{V}_{\mathrm{m}}=\sqrt{2} \mathrm{~V}_{\mathrm{RMS}}=\sqrt{2} \mathrm{~V}_{\mathrm{EFF}} \\
& \therefore \\
& \quad \mathrm{~V}_{\mathrm{p}}=\sqrt{2} \mathrm{~V}_{\mathrm{RMS}}=\sqrt{2}(115)=163 \mathrm{~V}
\end{aligned}
$$

According the Eq. 1.5:

$$
\begin{gathered}
\mathrm{I}_{\mathrm{p}}=\mathrm{I}_{\mathrm{m}}=\sqrt{2} \mathrm{I}_{\mathrm{RMS}}=\sqrt{2} \mathrm{I}_{\mathrm{EFF}} \\
\therefore \mathrm{I}_{\mathrm{p}}=\sqrt{2} \mathrm{I}_{\text {RMS }}=\sqrt{2}(5)=7.1 \mathrm{~A}
\end{gathered}
$$

2. A phase conductor of a transmission line is one mile long and has a diameter of 1.5 inch. The conductor is composed of aluminum. Calculate the electrical resistance of this conductor.

## Solution:

Solution Strategy: Since the resistivity value of aluminum, as stated in Segment 1, is in metric or SI unit system, the length and diameter specifications stated in this problem must be streamlined in metric units before application of Eq. 1.9 for determination of resistance in ohms $(\Omega \mathrm{s})$.
$\mathbf{L}=1$ mile $=1,609 \mathrm{~m}$
Diameter $=1.5$ inch $=0.0381 \mathrm{~m} ; \therefore \mathbf{R}=$ Radius $=\mathrm{D} / 2=0.019 \mathrm{~m}$
$\mathbf{A}=$ Area of cross-section $=\boldsymbol{\pi} \cdot \mathbf{R}^{\mathbf{2}}=(3.14) \cdot(0.019)^{2}=0.00113 \mathrm{~m}^{2}$
$\rho_{\text {aluminum }}=28.2 \mathrm{n} \Omega \mathrm{m}=28.2 \times 10^{-9} \Omega \mathrm{~m}$; given in Segment 1

$$
\begin{align*}
\mathrm{R} & =\rho \cdot \frac{\mathrm{L}}{\mathrm{~A}}  \tag{Eq. 1.9}\\
\therefore \quad \mathrm{R} & =\rho \cdot \frac{\mathrm{L}}{\mathrm{~A}}=28.2 \times 10^{-9}\left(\frac{1609}{0.00113}\right)=0.039 \Omega
\end{align*}
$$

3. What is the resistance of the following circuit as seen from the battery?


## Solution:

No current will flow through the two $4 \Omega$ resistors, the two $3 \Omega$ resistors, or the $7 \Omega$ resistor. The current finds the path of least resistance through the highlighted short circuit segment of the circuit. Therefore, the circuit reduces to one $6 \Omega$ in series with two $12 \Omega$ in parallel.

$$
\begin{aligned}
& \mathrm{R}=6 \Omega+\frac{(12 \Omega) \cdot(12 \Omega)}{(12 \Omega+12 \Omega)} \\
& \mathrm{R}=6 \Omega+6 \Omega=12 \Omega
\end{aligned}
$$

4. Consider the RC circuit shown in the diagram below. The source voltage is 12 V . The capacitor voltage is 2 V before the switch is closed. The switch is closed at $\mathrm{t}=0$. What would the capacitor voltage be at $\mathrm{t}=5 \mathrm{sec}$ ?


## Solution:

This particular case represents a capacitor charging scenario. Given the value of $\mathbf{R}, \mathbf{C}, \boldsymbol{v} \mathbf{c}(\mathbf{0})$ and the source voltage $\boldsymbol{V}$, Equation 1.18 allows us to calculate the voltage after and elapsed time " $t$," during the capacitor charging phase.

$$
v_{c}(t)=v_{c}(0) e^{-\frac{t}{R C}}+V\left(1-e^{-\frac{t}{R C}}\right)
$$

Eq. 1.18

## Given:

$$
\begin{aligned}
& \mathbf{R}=1 \mathrm{M} \Omega=1,000,000 \Omega \\
& \mathrm{C}=5 \mu \mathrm{~F}=5 \times 10^{-6} \mathrm{~F} \\
& \boldsymbol{v}_{\mathbf{c}}(\mathbf{(})=2 \mathrm{~V}=\text { Voltage across the capacitor at } \mathrm{t}=0 \\
& v_{\mathrm{c}}(\mathbf{t})=\text { Voltage across the capacitor, at a given time } \mathbf{t}=\text { ? } \\
& \boldsymbol{V}=\text { Voltage of the power source }=12 \mathrm{~V} \\
& \mathbf{t}=5 \mathrm{sec} \\
& \boldsymbol{\tau}=R C \text { circuit time constant }=\boldsymbol{R} . \boldsymbol{C} \\
& =(1,000,000)\left(5 \times 10^{-6}\right)=5 \mathrm{sec}
\end{aligned}
$$

Substitute the given and derived values in Eq. 1.18:

$$
\begin{aligned}
v_{c}(t) & =v_{c}(0) \cdot e^{-\frac{t}{R C}}+12 \cdot\left(1-e^{-\frac{t}{R C}}\right) \\
& =(2 V) \cdot\left(e^{-\frac{5}{5}}\right)+(12) \cdot\left(1-e^{-\frac{5}{5}}\right) \\
& =(2 \mathrm{~V}) \cdot(0.368)+(12) \cdot(0.632) \\
& =0.736 \mathrm{~V}+7 \cdot 585 \mathrm{~V} \\
& =8.32 \mathrm{~V}
\end{aligned}
$$

5. Determine the equivalent capacitance for the DC circuit shown in the circuit diagram below if $\mathrm{C}_{1}=5 \mu \mathrm{~F}$ and $\mathrm{C}_{2}=10 \mu \mathrm{~F}$.


## Solution:

Application of Eq. 1.21 to the two capacitor series circuit shown in the given circuit diagram yields:

$$
\begin{gathered}
C_{E Q}=\frac{C_{1} C_{2}}{C_{1}+C_{2}} \\
\begin{aligned}
& C_{E Q}=\frac{\left(5 \times 10^{-6}\right)\left(10 \times 10^{-6}\right)}{\left(5 \times 10^{-6}\right)+\left(10 \times 10^{-6}\right)} \\
& \quad=3.33 \times 10^{-6}=3.33 \mu \mathrm{~F}
\end{aligned}
\end{gathered}
$$

6. Determine the equivalent capacitance for the DC circuit shown below if this circuit consists of twenty $100 \mu \mathrm{~F}$ capacitors in series.


## Solution:

Apply Eq. 1.22 to " n " series capacitor circuit shown in diagram:

$$
\mathrm{C}_{\mathrm{EQ}}=\mathrm{C}_{\mathrm{EQ}-\mathrm{n}}=\frac{\mathrm{C}}{\mathrm{n}}
$$

Where, $\mathbf{n}=20$ and $\mathbf{C}=100 \mu \mathrm{~F}$

$$
\begin{aligned}
\mathrm{C}_{\mathrm{EQ}} & =\frac{100 \mu \mathrm{~F}}{20} \\
& =5 \mu \mathrm{~F}
\end{aligned}
$$

7. Determine the equivalent capacitance in series and parallel combination circuit shown below. The capacitance values are: $\mathrm{C}_{1}=10 \mu \mathrm{~F}, \mathrm{C}_{2}=10 \mu \mathrm{~F}, \mathrm{C}_{3}=$ $20 \mu \mathrm{~F}, \mathrm{C}_{4}=20 \mu \mathrm{~F}$.


## Solution:

The capacitors in this circuit that lend themselves to linear combination are $\mathrm{C}_{3}$ and $\mathrm{C}_{4}$. Therefore, the combined capacitance, $\mathrm{C}_{34}$, would be:

$$
\mathrm{C}_{34}=\mathrm{C}_{3}+\mathrm{C}_{4}=20 \mu \mathrm{~F}+20 \mu \mathrm{~F}=40 \mu \mathrm{~F}
$$

Then, by applying Eq. 1.24 to this special hybrid capacitor combination case:

$$
\begin{aligned}
C_{E Q}= & \frac{C_{1} C_{2} C_{34}}{C_{1} C_{2}+C_{2} C_{34}+C_{1} C_{34}} \\
C_{E Q} & =\frac{\left(10 \times 10^{-6}\right)\left(10 \times 10^{-6}\right)\left(40 \times 10^{-6}\right)}{\left(10 \times 10^{-6}\right)\left(10 \times 10^{-6}\right)+\left(10 \times 10^{-6}\right)\left(40 \times 10^{-6}\right)+\left(10 \times 10^{-6}\right)\left(40 \times 10^{-6}\right)} \\
\mathrm{C}_{\mathrm{EQ}} & =4.44 \mu \mathrm{~F}
\end{aligned}
$$

8. Assume that the circuit in problem 6 is powered by a 60 Hz AC source instead of the DC source. Determine the total capacitive reactance, $X_{c}$, seen by the AC source.

## Solution:

If the DC source is replaced by an AC source, the circuit would appear as follows:


As computed in problem 6, the combined or net capacitance contributed to the circuit by the parallel and series network of capacitors is $\mathrm{C}_{\mathrm{EQ}}=4.44 \mu \mathrm{~F}$. Then, by applying Eq.1.26:

$$
\mathrm{X}_{c}=\frac{1}{\omega \mathrm{C}}=\frac{1}{2 \pi \mathrm{fC}}
$$

Where,

$$
\begin{gathered}
\mathrm{f}=\text { frequency }=60 \mathrm{~Hz} \\
\mathrm{C}_{\mathrm{EQ}}=4.44 \mu \mathrm{~F} \\
\mathrm{X}_{c}=\frac{1}{\omega \mathrm{C}}=\frac{1}{2 \pi \mathrm{fC}}=\frac{1}{2(3.14)(60)\left(4.44 \times 10^{-6}\right)}=597 \Omega
\end{gathered}
$$

9. Consider the series RL circuit shown in the diagram below. The source voltage is $12 \mathrm{~V}, \mathrm{R}=10 \Omega$ and $\mathrm{L}=10 \mathrm{mH}$. The switch is closed at $\mathrm{t}=0$. What would be magnitude of current flowing through this circuit at $\mathrm{t}=2 \mathrm{~ms}$ ?


## Solution:

In most series RL cases, the current value at a certain time " $t$ " can be predicted through Eq. 1.31.

$$
i_{L}(t)=i_{R}(t)=i(0) e^{-\frac{R}{L} t}+\frac{V}{R}\left(1-e^{-\frac{R}{L} t}\right)
$$

## Given:

$$
\begin{aligned}
\mathrm{t} & =2 \times 10^{-3} \mathrm{~s} \\
\mathrm{~L} & =10 \times 10^{-3} \mathrm{H} \\
\mathrm{R} & =10 \Omega \\
\mathrm{~V} & =12 \mathrm{~V} \\
\mathrm{i}(0) & =0 \\
i_{L}(t) & =(0) e^{-\frac{10}{0.01}(0.002)}+\frac{12}{10}\left(1-e^{-\frac{10}{0.01}(0.002)}\right) \\
i_{L}(t) & =\frac{12}{10}\left(1-e^{-\frac{10}{0.01}(0.002)}\right) \\
i_{L}(t) & =(1.2)(1-0.135)=1.04 \mathrm{~A}
\end{aligned}
$$

10. Consider the series RL circuit given in problem 9, in discharge mode, with voltage source removed. Parameters such as $\mathrm{R}=10 \Omega$ and $\mathrm{L}=10 \mathrm{mH}$, are the same. The switch has been closed for long period of time, such that the current has developed to the maximum or steady state level 1.04 A. How much time would need to elapse for the current to drop to 0.5 A after the switch is opened.


## Solution:

Apply series RL current equation, Eq. 1.31.

$$
i_{L}(t)=i_{R}(t)=i(0) e^{-\frac{R}{L} t}+\frac{V}{R}\left(1-e^{-\frac{R}{L} t}\right)
$$

## Given:

$$
\begin{aligned}
\mathrm{t} & =? \\
\mathrm{~L} & =10 \times 10^{-3} \mathrm{H} \\
\mathrm{R} & =10 \Omega \\
\mathrm{~V} & =0 \\
\mathrm{i}(0) & =1.04 \mathrm{~A} \\
\mathrm{i}_{\mathrm{L}}(\mathrm{t}) & =0.5 \mathrm{~A} \\
i_{L}(t) & =(0.5)=(1.04) e^{-\frac{10}{0.01} t}+(0)\left(1-e^{-\frac{10}{0.01} t}\right) \\
0.5 & =(1.04) e^{-\frac{10}{0.01} t} \\
0.481 & =e^{-\frac{10}{0.01} t}
\end{aligned}
$$

$$
\ln (0.481)=\ln \left(e^{-\frac{10}{0.01} t}\right)
$$

$$
-.7324=-1000 t
$$

$$
\mathrm{t}=0.00073 \mathrm{~s} \text { or } 0.73 \mathrm{~ms}
$$

11. Determine the equivalent inductance $\mathrm{L}_{\mathrm{EQ}}$ for three parallel inductor DC circuit shown in the diagram below if $\mathrm{L}_{1}=2 \mathrm{mH}$, and $\mathrm{L}_{2}=5 \mathrm{mH}$ and $\mathrm{L}_{3}=$ 20 mH .


## Solution:

Apply Eq. 1.36 to compute $\mathrm{L}_{\mathrm{EQ}}$ for the three parallel inductor circuit.

$$
\mathrm{L}_{\mathrm{EQ}}=\frac{\mathrm{L}_{1} \mathrm{~L}_{2} \mathrm{~L}_{3}}{\mathrm{~L}_{1} \mathrm{~L}_{2}+\mathrm{L}_{2} \mathrm{~L}_{3}+\mathrm{L}_{1} \mathrm{~L}_{3}}
$$

EE Fundamentals and DC © ; $(23 \mathrm{mH})(5 \mathrm{mH})(20 \mathrm{mH})$
$(2 \mathrm{mH})(5 \mathrm{mH})+(5 \mathrm{mH})(20 \mathrm{mH})+(2 \mathrm{mH})(20 \mathrm{mH})$
$=1.33 \mathrm{mH}$
12. Calculate the net or total inductance as seen from the 24 V source vantage point in the circuit shown below.


## Solution:

Focus on the parallel combination of $\mathbf{L}_{\mathbf{2}}, \mathbf{L} \mathbf{3}$, and $\mathbf{L}_{\mathbf{4}}$, first. Apply Eq. 1.36 to calculate the equivalent inductance $\mathbf{L} \mathbf{2 3 4}$ for the three parallel inductors:

$$
\begin{aligned}
\mathrm{L}_{234} & =\frac{\mathrm{L}_{2} \mathrm{~L}_{3} \mathrm{~L}_{4}}{\mathrm{~L}_{2} \mathrm{~L}_{3}+\mathrm{L}_{3} \mathrm{~L}_{4}+\mathrm{L}_{1} \mathrm{~L}_{4}} \\
& =\frac{(20 \mathrm{mH})(30 \mathrm{mH})(40 \mathrm{mH})}{(20 \mathrm{mH})(30 \mathrm{mH})+(30 \mathrm{mH})(40 \mathrm{mH})+(20 \mathrm{mH})(40 \mathrm{mH})} \\
& =9.23 \mathrm{mH}
\end{aligned}
$$

This reduces the circuit as shown below:


Inductors $L_{1}$ and $L_{234}$, in this reduced circuit, lend themselves to a linear combination. Therefore, the equivalent inductance $\mathbf{L}_{E Q}$ for the entire parallel and series inductor hybrid circuit would be:

$$
\mathrm{L}_{E Q}=\mathrm{L}_{1}+\mathrm{L}_{234}=10 \mathrm{H}+9.23 \mathrm{H}=19.23 \mathrm{H}
$$

13. Assume that the circuit in Problem 12 is powered by a 60 Hz AC source. Calculate the inductive reactance, $\mathrm{X}_{\mathrm{L}}$, as seen by the AC voltage source.

## Solution:

If the DC source is replaced by an AC source, the circuit would appear as follows:

$\mathrm{L}_{\mathrm{EQ}}$, as seen by the AC voltage source, is shown in the simplified equivalent circuit below:


As computed in Problem 12, the combined or net inductance contributed to the circuit by the parallel and series network of inductors is $\mathrm{L}_{\mathrm{EQ}}=19.23 \mathrm{mH}$. Then, by applying Eq. 1.37 , the inductive reactance, $\mathbf{X}_{\mathbf{L}-E Q}$ as seen by the AC voltage source $\mathrm{V}_{\mathrm{AC}}$, would be:

$$
\begin{aligned}
X_{L-E Q} & =\omega \cdot L=(2 \pi f) \cdot L_{E Q} \\
& =2(3.14)(60 H z)(19.23 \mathrm{mH}) \\
& =7.25 \Omega
\end{aligned}
$$

## Segment 2 - Solutions

1. Determine the following for the DC circuit shown below if $\mathrm{R}_{1}=5 \Omega, \mathrm{R}_{2}=$ $\mathrm{R}_{3}=10 \Omega$, and $\mathrm{R}_{4}=\mathrm{R}_{5}=20 \Omega$ :
a) Current flowing through resistor $\mathrm{R}_{1}$
b) Voltage across resistor $\mathrm{R}_{5}$


## Solution:

a) $\mathbf{R}_{\text {eq }}$ was derived in Example 1.2 as follows:

Combination of $\mathrm{R}_{2}$ and $\mathrm{R}_{4}=\mathrm{R}_{2,4}=\mathrm{R}_{2}+\mathrm{R}_{4}=10 \Omega+20 \Omega=30 \Omega$
Combination of $\mathrm{R}_{3}$ and $\mathrm{R}_{5}=\mathrm{R}_{3,5}=\mathrm{R}_{3}+\mathrm{R}_{5}=10 \Omega+20 \Omega=30 \Omega$
Combination of $\mathrm{R}_{2,4}$ and $\mathrm{R}_{3,5}=$

$$
\begin{gathered}
\mathrm{R}_{2-5}=\frac{(30 \Omega) \cdot(30 \Omega)}{(30 \Omega+30 \Omega)}=\frac{900}{60}=15 \Omega \\
\mathrm{R}_{\mathrm{eq}}=\mathrm{R}_{1}+\mathrm{R}_{2-5}=5 \Omega+15 \Omega=20 \Omega
\end{gathered}
$$

Current through $\mathrm{R}_{1}$ would be the same as the current through the 12 V supply:

$$
\mathrm{I}=\frac{\mathrm{V}}{\mathrm{R}_{\mathrm{eq}}}=\frac{12 \mathrm{~V}}{20 \Omega}=0.6 \mathrm{~A}
$$

b) One method for determining $\mathrm{V}_{\mathrm{R} 5}$, voltage across $\mathrm{R}_{5}$, is to first calculate $\mathrm{V}_{\mathrm{R} 2-5}$, the voltage across the combined resistance of resistances $\mathrm{R}_{2}, \mathrm{R}_{3}, \mathrm{R}_{4}$, and $\mathrm{R}_{5}$. Then, by applying voltage division, calculate $\mathrm{V}_{\mathrm{R} 5}$ :

According to Ohm's law:
$\mathrm{V}_{\mathrm{R} 2-5}=\mathrm{I} .\left(\mathrm{R}_{2-5}\right)=(0.6 \mathrm{~A}) \cdot(15 \Omega)=9 \mathrm{~V}$

Then, by applying the voltage division rule:

$$
\begin{aligned}
\mathrm{V}_{\mathrm{R} 5} & =(9 \mathrm{~V}) \cdot\left(\frac{\mathrm{R}_{5}}{\mathrm{R}_{5}+R_{3}}\right) \\
& =(9 \mathrm{~V}) \cdot\left(\frac{20 \Omega}{20 \Omega+10 \Omega}\right) \\
& =(9 \mathrm{~V}) \cdot(0.67)=6 \mathrm{~V}
\end{aligned}
$$

2. What is the current through the $6 \Omega$ resistor?


## Solution:

Simplify the circuit.
$3 \Omega$ in parallel with $6 \Omega=2 \Omega$
$2 \Omega$ in series with $4 \Omega=6 \Omega$

$$
i=\frac{6 \mathrm{~V}}{6 \Omega}=1 \mathrm{~A}
$$

$$
\begin{gathered}
R_{\text {parallel }}=3 \Omega \\
R_{\text {total }}=3 \Omega+6 \Omega=9 \Omega \\
i=(1 \mathrm{~A})\left(\frac{3 \Omega}{3 \Omega+6 \Omega}\right)=1 / 3 \mathrm{~A}
\end{gathered}
$$

3. Find the current through the $0.5 \Omega$ resistor.


## Solution:

The voltage sources around the left loop are equal to the voltage drops across the resistances.

$$
20 \mathrm{~V}-19 \mathrm{~V}=0.25 \Omega i_{1}+0.4 \Omega\left(i_{1}-i_{2}\right)
$$

The same is true for the right loop.

$$
19 \mathrm{~V}=0.4 \Omega\left(i_{2}-i_{1}\right)+0.5 \Omega i_{2}
$$

Solve two equations and for two unknowns, using the simultaneous equation method:

$$
\begin{aligned}
& 0.65 \Omega i_{1}-0.4 \Omega i_{2}=1 \mathrm{~V} \\
& -0.4 \Omega i_{1}+0.9 \Omega i_{2}=19 \mathrm{~V} \\
& i_{1}=20 \mathrm{~A} \\
& i_{2}=30 \mathrm{~A}
\end{aligned}
$$

The current through the $0.5 \Omega$ resistor is 30 A .
4. Determine the value of currents $\mathrm{I}_{1}, \mathrm{I}_{2}$ and $\mathrm{I}_{3}$ in the circuit shown below if the voltage source $\mathrm{V}_{3}$ fails in short circuit mode. The specifications of all components are listed in the table below:


| $\mathbf{R}_{\mathbf{1}}$ | $10 \Omega$ |
| :---: | :---: |
| $\mathbf{R}_{\mathbf{2}}$ | $2 \Omega$ |
| $\mathbf{R}_{\mathbf{3}}$ | $3 \Omega$ |
| $\mathbf{R}_{\mathbf{4}}$ | $4 \Omega$ |
| $\mathbf{R}_{\mathbf{5}}$ | $7 \Omega$ |
| $\mathbf{R}_{\mathbf{6}}$ | $3 \Omega$ |
| $\mathbf{R}_{\mathbf{7}}$ | $5 \Omega$ |
| $\mathbf{V}_{\mathbf{1}}$ | 20 V |
| $\mathbf{V}_{\mathbf{2}}$ | 5 V |
| $\mathbf{V}_{\mathbf{3}}$ | 12 V |

## Solution:

Two noteworthy observations are in order before formulation of the three equations Necessary for the derivation of the three unknown currents:
(1) Even though the stated value of voltage for source $V_{3}$ is 12 V , the value used in the formulation of second and third loop equations would be 0 V because, in this scenario, voltage source $\mathrm{V}_{3}$ is assumed to have failed in short circuit mode.
(2) As a matter of simplification, series resistors $R_{2}$ and $R_{7}$ are combined into one resistor $\mathrm{R}_{2-7}$

$$
\mathrm{R}_{2-7}=\mathrm{R}_{2}+\mathrm{R}_{7}=2 \Omega+5 \Omega=7 \Omega
$$

The revised, simplified, would then be:


The three simultaneous equations derived by applying KVL to loops 1, 2 and 3, as described in Example 2.5, are:

$$
\begin{aligned}
& 21 I_{1}+7 I_{2}-4 I_{3}=20 \\
& 7 I_{1}+13 I_{2}+3 I_{3}=5 \\
& -4 I_{1}+3 I_{2}+14 I_{3}=0
\end{aligned}
$$

As in Example 2.5, apply the Cramer's rule to solve for the three unknown currents $\mathrm{I}_{1}, \mathrm{I}_{2}$ and $\mathrm{I}_{3}$. The augmented matrix thus developed would be:

| 21 | 7 | -4 | 20 |
| :---: | :---: | :---: | :---: |
| 7 | 13 | 3 | 5 |
| -4 | 3 | 14 | 0 |

The coefficient matrix, denoted as $\mathbf{A}$, would be:
$\left|\begin{array}{ccc}21 & 7 & -4 \\ 7 & 13 & 3 \\ -4 & 3 & 14\end{array}\right|$

The determinant of the coefficient matrix, denoted as $|\mathbf{A}|$, would be:

$$
|\mathbf{A}|=21\{(13 \times 14)-(3 \times 3)\}-7\{(7 \times 14)-(-4 \times 3)\}-4\{(7 \times 3)-(4 \times 13)\}=\mathbf{2 5 7 1}
$$

The determinant of the substitutional matrix, $\mathbf{A}_{\mathbf{1}}$, for determining the value of $\mathrm{I}_{1}$, is denoted as $\left|\mathbf{A}_{\mathbf{1}}\right|$, and
$A_{1}=\left|\begin{array}{ccc}20 & 7 & -4 \\ 5 & 13 & 3 \\ 0 & 3 & 14\end{array}\right|$
$\left|\mathbf{A}_{1}\right|=20\{(13 \times 14)-(3 \times 3)\}-7\{(5 \times 14)-(0 \times 3)\}-4\{(5 \times 3)-(0 \times 13)\}=\mathbf{2 9 1 0}$

The determinant of the substitutional matrix, $\mathbf{A}_{2}$, for determining the value of $\mathrm{I}_{2}$, is denoted as $\left|\mathbf{A}_{\mathbf{2}}\right|$, and

$$
A_{2}=\left|\begin{array}{ccc}
21 & 20 & -4 \\
7 & 5 & 3 \\
-4 & 0 & 14
\end{array}\right|
$$

$\left|\mathbf{A}_{2}\right|=21\{(5 \times 14)-(0 \times 3)\}-20\{(7 \mathrm{x} 14)-(-4 \times 3)\}-4\{(7 \mathrm{x} 0)-(-4 \times 5)\}=\mathbf{- 8 1 0}$

The determinant of the substitutional matrix, $\mathbf{A}_{3}$, for determining the value of $\mathrm{I}_{3}$, is denoted as $\left|\mathbf{A}_{3}\right|$, and
$A_{3}=\left|\begin{array}{ccc}21 & 7 & 20 \\ 7 & 13 & 5 \\ -4 & 3 & 0\end{array}\right|$
$\left|\mathbf{A}_{\mathbf{3}}\right|=21\{(13 \mathrm{x} 0)-(3 \mathrm{x} 5)\}-7\{(7 \mathrm{x} 0)-(5 \mathrm{x}-4)\}+20\{(7 \mathrm{x} 3)-(-4 \mathrm{x} 13)\}=\mathbf{1 0 0 5}$

Applying the Cramer's rule, the unknown variables, currents $\mathbf{I}_{\mathbf{1}}, \mathbf{I}_{\mathbf{2}}$ and $\mathbf{I}_{\mathbf{3}}$, can be calculated by dividing the determinants of substitutional matrices $\mathbf{A}_{1}, \mathbf{A}_{2}$ and $\mathbf{A}_{\mathbf{3}}$, respectively, by the determinant of the coefficient matrix $\mathbf{A}$.

Therefore,

$$
\begin{aligned}
& \mathrm{I}_{1}=\frac{\left|\mathrm{A}_{1}\right|}{|\mathrm{A}|}=1.132 \mathrm{~A} \\
& \mathrm{I}_{2}=\frac{\left|\mathrm{A}_{2}\right|}{|\mathrm{A}|}=-0.315 \mathrm{~A} \\
& \mathrm{I}_{3}=\frac{\left|\mathrm{A}_{3}\right|}{|\mathrm{A}|}=0.391 \mathrm{~A}
\end{aligned}
$$

Note: The negative sign for $\mathbf{I}_{2}$ indicates that the counterclockwise direction assumed for this current is incorrect and that the correct direction of the flow of current in loop 2 is clockwise.
5. Use current division to determine the value of current $\mathrm{I}_{1}$ in the circuit below:


## Solution:

We must to determine the value of source current $\mathbf{I}$, first. In order to determine the value of current $\mathbf{I}$ flowing through the source and the two $5 \Omega$ resistors, we must consolidate all resistors into an equivalent resistance $\mathbf{R E Q}_{\mathrm{EQ}}$ and then apply the Ohm's law.

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{EQ}}=5 \Omega+\left(\frac{(10 \Omega) \cdot(10 \Omega)}{(10 \Omega)+(10 \Omega)}\right)+5 \Omega=5 \Omega+5 \Omega+5 \Omega=15 \Omega \\
& \mathrm{I}=\frac{\mathrm{V}}{\mathrm{R}_{\mathrm{EQ}}}=\frac{12 \mathrm{~V}}{15 \Omega}=0.8 \mathrm{~A}
\end{aligned}
$$

Apply current division formula in form of Eq. 2.7

$$
\mathrm{I}_{10 \Omega}=\mathrm{I}_{1}=\left(\frac{\mathrm{R}_{\text {parallel }}}{\mathrm{R}_{\text {total }}}\right) \cdot \mathrm{I}=\left(\frac{10 \Omega}{10 \Omega+10 \Omega}\right) \cdot(0.8 \mathrm{~A})=0 \cdot 4 \mathrm{~A}
$$

6. Using Kirchhoff's Voltage Law, calculate the current circulating in the series resistor network below:


## Solution:

This problem is similar to the Example 2.2, with following exceptions:

1) The two voltage sources are driving the current the same direction, i.e., clockwise.
2) There are four resistors in series instead of three.

Using the strategy described in Example 2.2 and preparing the circuit for KVL application, the circuit would appear as follows:


Apply the Ohm's law to define the voltages, or voltage drops, across the four resistors. Note: since all four of the resistors are in series, we could combine them into a single REQ before applying KVL. However, in this case we will keep resistors separate just to maintain consistency with the approach adopted in Example 2.2.

$$
\begin{aligned}
& \therefore \mathrm{V}_{\mathrm{R} 1}=\mathrm{IR}_{1}=10(\mathrm{I}), \quad \mathrm{V}_{\mathrm{R} 2}=\mathrm{IR}_{2}=20(\mathrm{I}), \\
& \quad \mathrm{V}_{\mathrm{R} 3}=\mathrm{IR}_{3}=10(\mathrm{I}), \text { and } \mathrm{V}_{\mathrm{R} 4}=\mathrm{IR}_{4}=30(\mathrm{I})
\end{aligned}
$$

With all voltages - voltage source, voltage load and voltage drops across the resistors - identified and their polarities noted, apply KVL by "walking" the annotated circuit beginning at the cathode or negative electrode of the voltage source, $\mathbf{V}_{1 \text { s }}$. Add all voltages, with respective polarities, as you make a complete circle around the circuit, in the clockwise direction.

$$
\begin{aligned}
& \Sigma \mathrm{V}=0 \\
& -18 \mathrm{~V}+10(\mathrm{I})+-6 \mathrm{~V}+20(\mathrm{I})+10(\mathrm{I})+30(\mathrm{I})=0 \\
& 70(\mathrm{I})=24 \mathrm{~V} \\
& \text { Or, } \mathrm{I}=\frac{24}{70}=0.343 \mathrm{~A}
\end{aligned}
$$

Ancillary exercise: Verify the derived value of current through the alternative, $\mathrm{R}_{\mathrm{EQ}}$ and the Ohm's law method, as illustrated in Approach 1 of the solution for Example 2.2.
7. Determine the value of voltage source current in the parallel circuit below using KCL, Kirchhoff's Current Law.
Ancillary question: If one of the $5 \Omega$ resistors is removed (or replaced with an open circuit) and the other one is replaced with a short circuit, what would be the source current?


## Solution:

KCL is applied to the given circuit after the node has been identified, circuit has been annotated with voltage designation, voltage polarity, branch currents and current directions. See circuit diagram below:


Subscribing to the definition of a node as a point where three or more conductors merge, the shaded segment in the diagram above is designated as the node for this circuit. Then, before applying KCL to determine the source
current - using the Ohm's law - define the individual currents through each of the resistors, in terms of the specific resistance values and the voltages around them:

$$
\mathrm{I}_{1}=\frac{\mathrm{V}_{1}}{\mathrm{R}_{1}} \quad \mathrm{I}_{2}=\frac{\mathrm{V}_{2}}{\mathrm{R}_{2}} \quad \mathrm{I}_{3}=\frac{\mathrm{V}_{3}}{\mathrm{R}_{3}} \quad \mathrm{I}_{4}=\frac{\mathrm{V}_{4}}{\mathrm{R}_{4}}
$$

Since all of the resistors are in parallel with the voltage source,

$$
\mathrm{V}_{1}=\mathrm{V}_{2}=\mathrm{V}_{3}=\mathrm{V}_{4}=\mathrm{V}_{\mathrm{s}}=20 \mathrm{~V}
$$

Therefore,

$$
\begin{aligned}
& I_{1}=\frac{20}{5}=4 A, \quad I_{2}=\frac{20}{3}=6.67 \mathrm{~A}, \quad I_{3}=\frac{20}{10}=2 \mathrm{~A} \\
& I_{4}=\frac{20}{5}=4 A
\end{aligned}
$$

Then, application of KCL at the designated node yields the following equation:

$$
\mathrm{I}=\mathrm{I}_{1}+\mathrm{I}_{2}+\mathrm{I}_{3}+\mathrm{I}_{4}
$$

Or,

$$
\mathrm{I}=4 \mathrm{~A}+6.67 \mathrm{~A}+2 \mathrm{~A}+4 \mathrm{~A}=16.67 \mathrm{~A}
$$

Ancillary Question: If one of the $5 \Omega$ resistors is open-circuited and the other one short-circuited, the parallel resistor network would appear as follows:


The highlighted segment in the circuit above represents the short circuit that replaces $\mathbf{R}_{1}$. Since $\mathbf{R}_{\mathbf{1}}$ is replaced by a short circuit - regardless of the disposition of other circuit elements - it becomes a path of least resistance for the entire circuit. In other words, the voltage source is short-circuited. Interpreted in terms of the Ohm's law, this would mean:

$$
\mathrm{I}=\frac{\mathrm{V}_{\mathrm{s}}}{0}=\infty
$$

Since infinite current is not practical, this means that a very large amount of current would pass through the shunt or short circuit resulting in a catastrophic failure (burning or melting) of the short circuiting conductor, the interconnecting wires or a fault in the voltage source.

## Appendix B

## Common Units and Unit Conversion Factors

## Power

In the SI or Metric unit system, DC power or "real" power is traditionally measured in watts and:

$$
\begin{aligned}
& \mathrm{kW}=1,000 \mathrm{Watts} \\
& \mathrm{MW}=1,000,000 \mathrm{~W} \text { atts }=10^{6} \mathrm{~W} \\
& \mathrm{GW}=1,000,000,000 \mathrm{Watts}=10^{9} \mathrm{~W} \\
& \mathrm{TW}=10^{12} \mathrm{~W}
\end{aligned}
$$

Where $\mathrm{k}=1000, \mathrm{M}=1000,000, \mathrm{G}=1$ billion, and $\mathrm{T}=1$ trillion.

Some of the more common power conversion factors that are used to convert between SI System and US system of units are listed below:

$$
\begin{aligned}
1.055 \mathrm{~kJ} / \mathrm{s}=1.055 \mathrm{~kW} & =1 \mathrm{BTU} / \mathrm{s} \\
1-\mathrm{hp}=\text { One hp } & =746 \mathrm{Watts} \\
& =746 \mathrm{~J} / \mathrm{s} \\
& =746 \mathrm{~N}-\mathrm{m} / \mathrm{s} \\
& =0.746 \mathrm{~kW} \\
& =550 \mathrm{ft}-\mathrm{lbf} / \mathrm{sec}
\end{aligned}
$$

## Energy

In the SI or Metric unit system, DC energy or "real" energy is traditionally measured in Wh, kWh, MWh, GWh, TWh ( $10{ }^{12} \mathrm{~Wh}$ ).

$$
\begin{aligned}
& \mathrm{kWh}=1,000 \text { Watt-hours } \\
& \mathrm{MWh}=1,000,000 \text { Watt-hour }=10^{6} \mathrm{~Wh} \\
& \mathrm{GWh}=1,000,000,000 \text { Watt-hours }=10{ }^{9} \mathrm{~Wh} \\
& \mathrm{TWh}=10^{12} \mathrm{~Wh}
\end{aligned}
$$

Some mainstream conversion factors that can be used to convert electrical energy units within the SI realm or between the SI and US realms are referenced below:

$$
\begin{aligned}
& 1000 \mathrm{~kW} \times 1 \mathrm{~h}=1 \mathrm{MWh} \\
& 1 \mathrm{BTU}=1055 \mathrm{~J}=1.055 \mathrm{~kJ} \\
& 1 \mathrm{BTU}=778 \mathrm{ft}-\mathrm{lbf}
\end{aligned}
$$

## Energy, Work and Heat Conversion Factors:

| Energy, Work or Heat |  |  |
| :---: | :---: | :---: |
| Btu | 1.05435 | kJ |
| Btu | 0.251996 | kcal |
| Calories (cal) | 4.184 | Joules (J) |
| $\mathrm{ft}-\mathrm{lbf}$ | 1.355818 | J |
| $\mathrm{ft}-\mathrm{lbf}$ | 0.138255 | $\mathrm{kgf}-\mathrm{m}$ |
| $\mathrm{hp}-\mathrm{hr}$ | 2.6845 | MJ |
| KWH | 3.6 | MJ |
| $\mathrm{m}-\mathrm{kgf}$ | 9.80665 | J |
| $\mathrm{~N}-\mathrm{m}$ | 1 | J |

## Power Conversion Factors:

| Power |  |  |
| :---: | :---: | :---: |
| Btu/hr | 0.292875 | Watt (W) |
| ft-lbf/s | 1.355818 | W |
| Horsepower <br> (hp) | 745.6999 | W |
| Horsepower | $550 . *$ | $\mathrm{ft}-\mathrm{lbf} / \mathrm{s}$ |

## Temperature Conversion Factors/Formulas:

| Temperature |  |  |
| :---: | :---: | :---: |
| Fahrenheit | $\left({ }^{\circ} \mathrm{F}-32\right) / 1.8$ | Celsius |
| Fahrenheit | ${ }^{\circ} \mathrm{F}+459.67$ | Rankine |
| Celsius | ${ }^{\circ} \mathrm{C}+273.16$ | Kelvin |
| Rankine | $\mathrm{R} / 1.8$ | Kelvin |

## Common Electrical Units, their components and nomenclature:

| Force | Newton | $\mathbf{N}$ | $\mathbf{k g ~ m ~ s}^{-2}$ |
| :--- | :---: | :---: | :---: |
| Energy | joule | $\mathbf{J}$ | $\mathbf{k g ~ m}^{2} \mathbf{s}^{-2}$ |
| Power | watt | $\mathbf{W}$ | $\mathbf{k g ~ m}^{2} \mathbf{s}^{-3}$ |
| Frequency | hertz | Hz | $\mathbf{s}^{-1}$ |
| Charge | coulomb | $\mathbf{C}$ | $\mathbf{A ~ s}$ |
| Capacitance | farad | $\mathbf{F}$ | $\mathbf{C}^{2} \mathbf{s}^{2} \mathbf{~ k g}^{-1}$ <br> $\mathbf{m}^{-2}$ |
| Magnetic <br> Induction | tesla | $\mathbf{T}$ | $\mathbf{k g ~ A}^{-1} \mathbf{s}^{-2}$ |

## Common Unit Prefixes:

| $1.00 \mathrm{E}-12$ | pico | p |
| :---: | :---: | :---: |
| $1.00 \mathrm{E}-09$ | nano | n |
| $1.00 \mathrm{E}-06$ | micro | $\mu$ |
| $1.00 \mathrm{E}-03$ | milli | m |
| $1.00 \mathrm{E}+03$ | kilo | k |
| $1.00 \mathrm{E}+06$ | mega | M |
| $1.00 \mathrm{E}+09$ | giga | G |
| $1.00 \mathrm{E}+12$ | tera | T |

## Wire Size Conversions:

A circular mil can be defined as a unit of area, equal to the area of a circle with a diameter of one mil (one thousandth of an inch), depicted as:

1 circular mil is approximately equal to:

- 0.7854 square mils ( 1 square mil is about 1.273 circular mils)
- $7.854 \times 10^{-7}$ square inches ( 1 square inch is about 1.273 million circular mils)
- $5.067 \times 10^{-10} \mathrm{~m}^{2}$
- $506.7 \mu^{2}$

1000 circular mils $=1 \mathrm{MCM}$ or 1 kcmil , and is (approximately) equal to:

- $\quad 0.5067 \mathrm{~mm}^{2}$, so $2 \mathrm{kcmil} \approx 1 \mathrm{~mm}^{2}$


## AWG to Circular Mil Conversion

The formula to calculate the circular mil for any given AWG (American Wire Gage) size is as follows:
$\boldsymbol{A}_{\boldsymbol{n}}$ represents the circular mil area for the AWG size $\boldsymbol{n}$.

$$
A_{n}=\left(5 \times 92^{\frac{36-n}{39}}\right)^{2}
$$

For example, a AWG number 12 gauge wire would use $\mathbf{n}=\mathbf{1 2}$; and the calculated result would be 6529.946789 circular mils

Circular Mil to $\mathbf{~ m m}^{\mathbf{2}}$ and Diameter ( $\mathbf{m m}$ or in) Conversion:

| kcmil or, | $\mathbf{m m}^{2}$ | Diameter |  |
| :---: | :---: | :---: | :---: |
|  |  |  | in. |
|  |  | $\mathbf{m m}$ |  |
| 250 | 126.7 | 0.5 | 12.7 |
| 300 | 152 | 0.548 | 13.91 |
| 350 | 177.3 | 0.592 | 15.03 |
| 400 | 202.7 | 0.632 | 16.06 |
| 500 | 253.4 | 0.707 | 17.96 |
| 600 | 304 | 0.775 | 19.67 |
| 700 | 354.7 | 0.837 | 21.25 |
| 750 | 380 | 0.866 | 22 |
| 800 | 405.4 | 0.894 | 22.72 |
| 900 | 456 | 0.949 | 24.1 |
| 1000 | 506.7 | 1 | 25.4 |
| 1250 | 633.4 | 1.118 | 28.4 |
| 1500 | 760.1 | 1.225 | 31.11 |
| 1750 | 886.7 | 1.323 | 33.6 |
| 2000 | 1013.4 | 1.414 | 35.92 |

## Appendix C - Greek Symbols Commonly Used in Electrical

 Engineering

